values of ε and H_0, the point of inflection G moves along the free surface (according to the results of Sect.2, $g > 1$ in this case) and subsequently passes onto the line of separation (then $g < 1$). In the example cited in Fig.1, the point of inflection G is located on the line of separation and has the coordinates $x = 2.128$ and $y = -0.961$.

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ESTIMATES OF THE FLOW RATE CHARACTERISTICS IN THE THEORY OF FILTRATION AND HEAT CONDUCTION*

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In developing the approach proposed in /1, 2/, it is shown that it is possible to obtain estimates of the flow rate characeristics in the case of spatial, stationary linear filtration of an incompressible fluid in an inhomogeneous porous medium. The volume of the filtration domain and the area of a segment of a boundary of indeterminate form are employed as the decisive geometric characteristics (in the planar case, it is the area of the domain and the length of a segment of the boundary of indeterminate form). The corresponding boundary value problems are formulated. The subdomains of the domain of existence of a solution in which the extremal estimate is a lower estimate are indicated. An example is given.

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The description is presented in terms of filtration theory. In view of the known analogy between linear filtration and conductive heat transfer, all the assertions and conclusions apertaining to the efficiency coefficient are transferred to the heat transfer coefficient.

1. Let us consider the steady state filtration of a liquid in a domain G with a boundary $\partial G = \Gamma \cup \Gamma_1 \cup \Gamma_2$. In G, the pressure $p(x)$ satisfies the equation and the boundary conditions

$$\operatorname{div}(k \nabla p) = 0 \tag{1.1}$$

$$p(\mathbf{x}) = P,\ \mathbf{x} \in \Gamma_1;\ p(\mathbf{x}) = 0,\ \mathbf{x} \in \Gamma_2;\ \partial p/\partial n = 0,\ \mathbf{x} \in \Gamma \tag{1.2}$$

Here k is the filtration coefficient, P is the pressure gradient at the inlet and the outlet of the filtration flow $(P > 0)$ and $\mathbf{n}$ is the external normal unit vector with respect to the domain G.

Let a certain part of the boundary of the domain have an indeterminate configuration. We shall denote the classes of domains having such a segment on a surface of constant pressure of an impermeable surface by Φ and Ψ respectively. Let γ be a smooth surface, without points of selfintersection, which corresponds to a certain configuration of the segment which is being varied and let γ^* be a non-smooth surface which is not, generally speaking, very dissimilar from it. The system of local orthogonal coordinates (ξ, η) is introduced on γ. The position of γ^* with respect to γ is specified analytically by means of a continuous function $\delta n(\xi, \eta)$ of the algebraic mangitude of the vector of the increment along the normal to γ. Let us agree to assume that $\delta n > 0$ if the above-mentioned vector is identical in direction to the external normal to the domain G. We shall say that the surface γ and γ^* are close if one of the two conditions

$$|\delta n| < \delta,\ |\nabla \delta n(\xi, \eta)| < \delta;\ |\delta n| < \delta,\ |\nabla\, \delta n(\xi, \eta)| < U \tag{1.3}$$

is satisfied, where the magnitude of U is bounded and δ is small (in particular, less than the minimum radius of curvature of the surface γ). Surfaces which satisfy the first of conditions (1.3) will also satisfy the second so that the latter defines a wider class of surfaces γ^* which will also be implied everywhere in the ensuing discussion with the exception of cases where something is specially said to the contrary.

We also assume that one of the two conditions is satisfied on the edges of the surface γ: either the surface γ is perpendicular to the surface to which it is contiguous or $\delta n(\xi, \eta)$ at such points is equal to zero. Then, γ^* together with a specified part of the boundary ∂G forms a closed surface without points of selfintersection and thereby defines a certain physical flow domain G^*. We denote the pressure distribution function, which satisfies Eq.(1.1) in this domain and the boundary conditions (1.2), by $p^*(\mathbf{x})$.

Let us now determine the general form of the increment in the efficiency coefficient C as a function of the change in the configuration of a segment of γ.

Assertion 1. Under the assumptions which have been made above, a variation of the functional $C[\gamma]$ in the class of domains Φ can be represented in the following form:

$$\delta C = -\int_\gamma k \left|\frac{\nabla p}{P}\right|^2 \left[\delta n - \left(\varkappa + \frac{\partial \ln k}{\partial n}\right) \delta n^2\right] d\sigma + \int_{G^*} k \left|\frac{\nabla \delta p}{P}\right|^2 dV + o(\delta^2), \quad \delta p = p^*(\mathbf{x}) - p(\mathbf{x}) \tag{1.4}$$

where $\varkappa$ is the curvature of an arc of γ in the case of planar flow or the mean curvature of the surface γ in the case of spatial flow.

We shall present the proof for spatial flow. By analogy with /1/, we introduce the auxiliary functional

$$J[\gamma] = \int_G k\, |\nabla p|^2\, dV \tag{1.5}$$

where $p(\mathbf{x})$ satisfies the boundary value Problem (1.1), (1.2). By applying Green's formula to it and taking account of (1.2), it can be shown that $J[\gamma] = P^2 C[\gamma]$. It is therefore sufficient to determine the form of the variation in the functional (1.5) while preserving the pressure differential.

Let the function $p(\mathbf{x})$ be analytically extendable through the surface γ and, consequently, satisfy Eq.(1.1) everywhere in the domain G^*. The above-mentioned variation in the functional (1.5) is then representable in the form ($R = (G^* \setminus G) \cup (G \setminus G^*)$ subsequently)

$$(\delta J)_p = \pm \int_R k \mid \nabla p \mid^2 dV - \int_{G^*} k \mid \nabla \delta p \mid^2 dV + 2 \int_{G^*} k \nabla p^* \nabla \delta p \, dV \tag{1.6}$$

Here and below, the plus sign refers to the domain $G^* \setminus G$ and the minus sign to the domain $G \setminus G^*$. By applying Green's formula to the last integral in (1.6), we find that it will be equal to $2\,(\delta J)_p$. Consequently,

$$(-\delta J)_p = \pm \int_R k \mid \nabla p \mid^2 dV - \int_{G^*} k \mid \nabla \delta p \mid^2 dV \tag{1.7}$$

We now introduce the rectilinear coordinate ζ which is measured from γ in the direction of the normal n. It is obvious that the set (ξ, η, ζ) forms a local orthogonal coordinate system. The non-zero components of the metric tensor in this system will have the form

$$g_{11} = g_{11}^\circ (1 + \varkappa_1 \zeta)^2, \quad g_{22} = g_{22}^\circ (1 + \varkappa_2 \zeta)^2, \quad g_{33} = 1 \tag{1.8}$$

where $\varkappa_1$ and $\varkappa_2$ are the curvatures of the lines $\eta = \text{const}$ and $\xi = \text{const}$ on the surface γ and g_{11}° and g_{22}° are the components of the metric tensor on the surface γ which satisfy its internal geometry /3/.

By writing the differential operator (1.1) in this system of coordinates we find that $\partial \mid \nabla p \mid / \partial \zeta = - \mid \nabla p \mid (\varkappa + \partial \ln k / \partial \zeta)$ on γ, where $\varkappa = \varkappa_1 + \varkappa_2$ is the mean curvature of the surface γ.

We shall assume that the functions $k(\mathbf{x})$ and $\mid \nabla p(\mathbf{x}) \mid$ are analytical with respect to the variable ζ in the neighbourhood of γ and, consequently, can be expanded in a Taylor series. Allowing for this and also the fact that the equality $dV = (1 + \varkappa_1 \zeta)(1 + \varkappa_2 \zeta)\, d\sigma \mid d\zeta \mid$ is valid in the neighbourhood of γ. we integrate the first two integrals in (1.7) with respect to ζ from 0 to δn after which we are convinced of the validity of formula (1.4).

Remark 1. If a variation of the functional $C[\gamma]$ is considered in the class of domains Ψ, then the last integral in (1.6) will be equal to zero. Hence, in this case, a variation in the functional (1.5) has a form which only differs from (1.7) in that it has an opposite sign. All the remaining arguments in the proof remain in force and the final form of the incremental growth in the efficiency coefficient in this case only differs from (1.4) in that it has the opposite sign.

An integral over the domain G^* occurs in the second variation of the functional $C[\gamma]$. This integral cannot be expressed in terms of δn and the parameters of the flow corresponding to γ. Meanwhile, it is possible to obtain certain estimates.

Assertion 2. The following estimate holds for the integral over the domain G^* occurring in (1.4):

$$\int_{G^*} k \left| \frac{\nabla \delta p}{P} \right|^2 dV \geqslant \int_\gamma k \left| \frac{\nabla p}{P} \right|^3 \delta n^2 \, d\sigma \tag{1.9}$$

Proof. Let us subdivide the flow corresponding to the configuration γ^* by means of impermeable surfaces into a set of thin tubes of flow which correspond to the flow with the configuration of γ of the part which is being varied. The efficiency coefficient of this flow C', which is equal to the sum of the efficiency coefficients of the flows in each of the tubes taken individually, will not be greater than the real coefficient C^* /4/. Let us assign a number to each flow tube and consider, for example, the i-th tube.

Let G_i and G_i^* be parts of the domains G and G^* and let γ_i and γ_i^* be segments of the surfaces γ and γ^* which are intersected by the tube; $\Delta\sigma_i$ and $\Delta\sigma_i^*$ are the areas of these segments, $p_i(\mathbf{x})$ and $p_i^*(\mathbf{x})$ are the pressures and C_i and C_i^* are the efficiency coefficients of the domains G_i and G_i^* respectively. By making use of Assertion 1 and taking account of the smallness of $\Delta\sigma_i$, we find

$$\delta C_i = -k \left| \frac{\nabla p}{P} \right|^2 \left[\delta n - \left(\varkappa + \frac{\partial \ln k}{\partial n} \right) \frac{\delta n^2}{2} \right] \Big|_{\gamma_i} \triangle\sigma_i + \frac{k}{P^2} \delta p_i \delta \left(\frac{\partial p_i}{\partial n} \right) \Big|_{\gamma_i^*} \triangle\sigma_i^* + o(\delta^2) + o(\triangle\sigma_i) \tag{1.10}$$

The last term was obtained on passing from the corresponding integral in (1.4) over the domains G_i^* to a contour integral over ∂G_i^* and allowing for the boundary conditions.

Let us transform this term. We expand the function $p_i(\mathbf{x})$ in a Taylor series in the variable ζ and take account of the fact that, on account of the incompressibility of the liquid, $\delta(\partial p_i/\partial n)\Delta\sigma_i^* = -P\delta C_i$ on γ_i^*. By using expression (1.10) to within quantities of the order of δ, we get

$$\frac{k}{P^2}\delta p_i \delta\left(\frac{\partial p_i}{\partial n}\right)\bigg|_{\gamma_i^*}\Delta\sigma_i^* = k\left|\frac{\nabla p_i}{P}\right|^3 \delta n^2\bigg|_{\gamma_i}\Delta\sigma_i + o(\delta^2)$$

By substituting this relationship into (1.10) and summing over i, we find, in the limit when $\Delta\sigma_i \to 0$,

$$C' - C = \delta C - \int\limits_{G^*} k\left|\frac{\nabla\delta p}{P}\right|^2 dV + \int\limits_{\gamma} k\left|\frac{\nabla p}{P}\right|^3 \delta n^2\, d\sigma + o(\delta^2)$$

Allowing for the fact that $\delta C \geqslant C' - C$, we compare the last expression with (1.4), whereupon we are convinced of the validity of the estimate (1.9).

By fixing one or the other of the integral geometric characteristics of the domain and thereby specifying a certain subclass of domains from Φ or Ψ, one can treat isoperimetric problems on the extremum of the efficiency coefficient in this subclass. When account is taken of the estimate (1.9), the form of the second variation of the efficiency coefficient permits one in individual cases to judge the nature of the extremum.

2. Let us take the volume (or, in the case of a planar flow, the area) V of the filtration domain as the decisive geometrical characteristic. Using (1.8), an incremental growth in this volume, as in the functional of the configuration of a segment of γ, can be written in the form

$$\delta V = \pm\int\limits_{R}(1 + \varkappa_1\zeta)(1 + \varkappa_2\zeta)\, d\sigma\, |d\zeta|$$

After integration over ζ from 0 to δn we get

$$\delta V = \int\limits_{\gamma}\left(\delta n + \varkappa\frac{\delta n^2}{2}\right) d\sigma \tag{2.1}$$

Let us now consider the problem of the extremum of the efficiency coefficient in a class of domains Φ which have a fixed volume. This problem belongs to a number of isoperimetric problems which, as is well known, reduce to the problem of the absolute extremum of a certain functional. In the given case, this functional is $C + \lambda V$, where λ is an undetermined constant. The necessary condition for its extremum is that the first variation should be equal to zero and the sufficient condition is that, additionality, the second variation should be strongly positive. Allowing for this and starting out from (1.4) and (2.1), we arrive at the boundary value Problem (1.1), (1.2) with an additional condition on the unknown part of the boundary

$$k\,|P^{-1}\nabla p|^2 = \lambda,\ \mathbf{x} \in \gamma \tag{2.2}$$

If its solution exists and the configuration of γ found as a result satisfies the condition

$$\varkappa + |P^{-1}\nabla p| + \partial\ln k/\partial n > 0 \tag{2.3}$$

then a local minimum in the efficiency coefficient is ensured in the class of domains Φ having a fixed volume.

This problem for the planar case has been formulated in /1/. In /2/, a formulation of it has been given for a wide class of spatial filtration laws. It should be noted that, using the latter formulation on the class of domains with a fixed volume, an extremum of the functional is realized which, generally speaking, is not equal to the efficiency coefficient.

A more rigourous condition than (2.3) was obtained in /2/ when $k = \text{const}$ and there is a planar flow and class of planar problems was pointed out for which the boundary value Problem (1.1), (1.2), (2.2) is efficiently solved. Problems inolving a more complex flow geometry and an exponential filtration law have been considered in /5/.

Remark 2. If a problem on the extremum of the efficiency coefficient in a class of domains belonging to Ψ and having a fixed volume is considered then, by introducing the functional $C - \lambda V$, we also arrive at Problem (1.1), (1.2), (2.2) with an unknown segment of the boundary $\gamma \in \Gamma$. If a solution exists and the configuration of γ which is found as a result is convex everywhere with respect to the domain then, at the same time, a local maximum of the efficiency coefficient for the specified volume is realized (see Remark 1) and meanwhile a local minimum in the volume of the filtration domain is realized in view of the property of the interdependence of isoperimetric problems when the efficiency coefficient is fixed.

The proposal has been put forward in /4/ that "the estimte of the loss of petroleum throughout the volume of the limiting equilibrium pillars", which is found using a model for the filtration of a liquid with an initial gradient /6, 7/ is "an upper estimate". Remark 2 enables this to be substantiated. Let a planar domain G exist which has known boundaries from which a viscoplastic liquid (petroleum) is displaced by a viscous liquid (water) at $k = \text{const}$. Let us suppose that the flow was established and that a pillar of petroleum of unknown configuration remained close to some part or other of the impermeable boundary (Fig.1). In the above-mentioned model the boundary of the pillar is found from the conditions

$$|\nabla p| \geqslant \tau_0, \ \mathbf{x} \in G; \ |\nabla p| = \tau_0, \ \mathbf{x} \in \gamma \tag{2.4}$$

where γ is the boundary of the pillar and τ_0 is the initial gradient. It is obvious that the boundary value Problem (1.1), (1.2), (2.4) is identical to Problem (1.1), (1.2), (2.2). The additional condition from (2.4) which is imposed on $|\nabla p|$ in the domain G leads to the necessary condition of the convexity of the boundaries of the pillar with respect to the domain /8/. However, it then follows from Remark 2 that the solution of this problem realizes a local minimum of the area of the domain encompassed by the filtration flow when C is fixed and therefore enables one to estimate the loss in petroleum in the pillars from above using the known efficiency coefficient (here, τ_0 is a parameter which is determined during the course of solving the problem). It can be shown that the above-mentioned estimate also remains an upper estimate in the case of those variations in γ for which the points at which it comes into contact with the specified impermeable boundary move along the latter from the pillar and Remark 2 is now inapplicable.

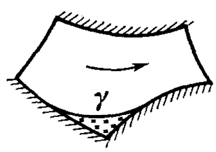

Fig.1

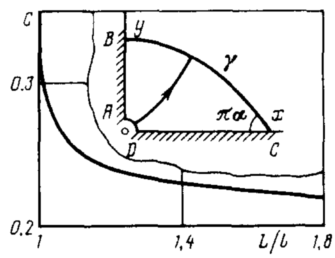

Fig.2

3. As the decisive geometric characteristic, let us now take the area (the length in the case of planar flow) Σ of the segment γ of the boundary of the domain which is being varied and find the form of the incremental change in Σ as the incremental change in the functional of the configuration of γ. By writing the Riemann metric $(dl)^2 = g_{11}(d\xi)^2 + g_{22}(d\eta)^2 + g_{33}(d\zeta)^2$ on the surface γ^* and allowing for the fact that, in this case, $\zeta = \delta n(\xi, \eta)$, we find the non-zero components of the metric tensor G_{ij}° which specifies its internal geometry in the local coordinates

$$\begin{gathered} G_{11}^\circ = g_{11} + g_{33}\delta n_\xi^2|_{\zeta=\delta n}, \quad G_{22}^\circ = g_{22} + g_{33}\delta n_\eta^2|_{\zeta=\delta n} \\ G_{12}^\circ = G_{21}^\circ = g_{33}\delta n_\xi \delta n_\eta|_{\zeta=\delta n} \end{gathered} \tag{3.1}$$

An incremental change in the area Σ of the segment γ which is being varied can be represented in the form /3/

$$\delta\Sigma = \int_\Omega \left[\sqrt{G_{11}^\circ G_{22}^\circ - G_{12}^\circ G_{21}^\circ} - \sqrt{g_{11}^\circ g_{22}^\circ}\right] d\xi\, d\eta$$

where Ω is the domain in the (ξ, η)-plane which corresponds to the segment γ of the boundary of the domain. Assuming that the first of conditions (1.3) is valid and making use of (3.1), we find

$$\delta\Sigma = \int_\gamma \left[\varkappa\delta n + \frac{1}{2}|\nabla\delta n(\xi, \eta)|^2\right] d\sigma + o(\delta^2) \tag{3.2}$$

Let us now consider the problem of the extremum of the efficiency coefficient in the class of domains from Φ which have the specified area (length) of the segment of the boundary which is being varied. By using arguments which are analogous to those employed in Sect.2 we arrive at the boundary value problem (1.1), (1.2) with an additional condition on the free segment of the boundary

$$k \mid P^{-1} \nabla p \mid^2 = \lambda \varkappa, \ \mathbf{x} \in \gamma \tag{3.3}$$

At the same time it may be asserted on the basis of (1.4) and (3.2) that, when Σ is fixed, the convexity of the free segment of the boundary is a sufficient condition for a minimum in C. In the case of the filtration of a liquid in a homogeneous porous medium the boundary value Problem (1.1), (1.2), (3.3) is mathematically equivalent to problems in the theory of a jet of an ideal fluid when capillary forces are taken into account. Efficient methods for solving them in the case of planar flows are described in /9, 10/.

As an example, let us consider the problem of the insulation of an infinitely long thin thermal conductor of circular cross-section of radius r, with a homogeneous material with a single thermal conductivity, where the surface of the conductor is symmetrical about the x- and y-axes and the problem is treated in the formulation (1.1), (1.2), (3.3). A quarter of the cross-section of the conductor and the insulation is shown in the upper part of Fig.2.

Let l be the width of the insulation AC and L be the length of the BC. In order to formulate the problem correctly, it is necessary to require that the point B should be a point of smoothness and that point C, generally speaking, should be a salient point where the tangent to the boundary is discontinuously rotated through a certain angle $2\pi\alpha$.

Let us first find the solution of the problem regarding the insulation of a point heat source (the boundary AD is contracted to the point A while the flow rate is maintained). We introduce the complex thermal flux potential $W = -t + i\psi$, the parametric complex variable u which varies in the semicircle $|u| < 1, \operatorname{Im} u \geqslant 0$ ($u = 1, -1, 0$ corresponds to the points A, B, C) and the auxiliary McLeod function /10/

$$\omega(z) = \frac{1}{\lambda} \int_{-1}^{z} \left(\frac{dW}{dz}\right)^2 dz + i$$

Allowing for the form of the domains over which ω and W vary, we find by means of conformal mappings

$$\frac{d\omega}{du} = \frac{2(1-\alpha)\sin(\pi\alpha)\, u^{\alpha}}{(u^{\alpha} - u)^2}, \quad \frac{dW}{du} = \frac{2q}{\pi\sqrt{u}\,(u-1)}$$

after which the geometry of the domain is recovered by quadrature. The link between the dimensionless length of the arc BC and the parameter α is defined by the formula

$$\frac{L}{l} = \frac{2\pi(1-\alpha) + \sin 2\pi\alpha}{2\,[\sin \pi\alpha + \pi(1-\alpha)\cos \pi\alpha]}, \quad 0 \leqslant \alpha \leqslant 1$$

whence it follows that the problem is solvable when $L/l \leqslant 2$.

Let us now return to the initial problem. An arc of a circle with its centre at the point $u = 1$ with a radius ρ which is small compared with unity

$$\rho = (r/l)[1 + \pi(1-\alpha)\operatorname{ctg} \pi\alpha](1-\alpha)^{-2} + o(r/l)$$

corresponds to the boundary AD in the domain of variation in u.

The required heat transfer coefficient is represented by the expression

$$C = (\pi/2)[\ln(\rho/4)^{-1} + O(r/l)]^{-1}$$

The dependence of C on L/l when $r/l = 10^{-3}$ is shown in Fig.2.

Let us now show how it is possible to find lower estimates of the heat transfer coefficient for the conductor which has been described in the case when the cross-section G of the insulation has an arbitrary configuration. One such estimates can be obtained by measuring the area of the domain G and determining the flow rate in a problem with the insulation arranged in the form of a circle of this area which is concentric with the conductor. In order to obtain a second estimate, we symmetrize the domain G with respect to two mutually perpendicular axes (while not increasing the thermal flux characteristics /4/). By taking the larger of the semi-axes of the resulting ellipse as the characteristic length l and determining a quarter of its perimeter using the graph in Fig.2, we find a lower estimate of the quantity C.

Let us now consider the case when the domain G is an ellipse and the conductor is located at its centre in greater detail. The problem of a flow from a pore of radius r to an elliptic supply contour corresponds to the filtration interpretation. To be specific, let the ratio of the semi-axes be equal to 0.1 and let us take the larger semi-axis as the characteristic distance l. Then, when $r/l = 10^{-3}$, we find /11/ the value of the coefficient

$C = 0.31$ and its lower estimate from the area of the ellipse $C \geqslant 0.27$. Using the graph in Fig.2, we obtain the estimate from the perimeter of the ellipse $C \geqslant 0.29$. This turned out to be more accurate. This is explained by the fact that, in addition to the integral characteristic of the domain (the perimeter), the width of the insulation was also taken into account in the second estimate.

When necessary, it is possible to improve the second estimate, if additional information concerning the length h of the segment AB (the upper part of Fig.2) is made of. In order to do this, the problem which has been considered should be solved in a more general formulation which treats B as a salient point at which the tangent is discontinuously rotated through a certain angle which is determined by specifying the quantity h.

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